# Arc-Jet Propulsion for Attitude and Orbit Control of an MORL

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**This study of the use of electrothermal arc-jet propulsion for control of the attitude and orbit of a manned orbiting research laboratory (MORL) is based on realistic, conservative mass estimates for the subsystems necessary for a 1-yr mission in a circular orbit at an altitude of 200 naut miles. Total impulse is based upon a continuous thrust requirement of 0.10 $lb_f$. The electrical energy required is supplied by a photovoltaic system. Other subsystems considered are momentum flywheels, propellants, and propellant storage. Hydrogen is the recommended propellant. With 14 engines, the total mass of the arc-jet system is approximately 7258 $lb_m$. By comparison, a stored-chemical propulsion system with a specific impulse of 300 sec has a mass of 11,790 $lb_m$. This represents a launch mass saving of 38% for the arc-jet system.**

## Nomenclature

$A$ = area, ft$^2$
$a$ = quantity defined by Eq. (11), $lb_m/lb_f$-sec; acceleration, ft/sec$^2$
$b$ = quantity defined by Eq. (14), $(lb_m/lb_f)^2$/sec
$C$ = correction factor, fixed mass independent of $I_{sp}$, drag coefficient
$D$ = drag force, $lb_f$
$E$ = solar radiation intensity, kw/ft$^2$
$e$ = quantity defined by Eq. (14), $lb_f$-sec/$lb_m{}^2$
$F$ = thrust, $lb_f$; tensile strength, $lb_f$/in.$^2$
$f$ = vertical load
$g$ = acceleration, ft/sec$^2$; proportionality constant, $lb_m$-ft/$lb_f$-sec$^2$
$H$ = enthalpy, Btu/$lb_m$
$h_{fg}$ = latent heat of vaporization, Btu/$lb_m$
$I_{sp}$ = specific impulse, $lb_f$-sec/$lb_m$
$J$ = mechanical equivalent of heat, ft-$lb_f$/Btu
$K$ = thermal conductivity, Btu/hr-ft-°R
$M$ = mass, $lb_m$
$\dot{m}$ = mass flow rate, $lb_m$/sec
$N$ = number of support rods
$n$ = material parameter
$P$ = power, kw; pressure, $lb_f$/in.$^2$
$\dot{Q}$ = heat energy required per unit time to boil off propellant, Btu/hr
$\dot{q}$ = heat flow per unit time, Btu/hr
$R$ = mean radius of earth, ft
$r$ = radius, ft
$S$ = safety factor
$T$ = temperature, °$R$
$t$ = time, sec
$V$ = velocity, fps; volume, ft$^3$
$\alpha$ = specific mass of power supply
$\beta$ = a factor to account for propellant storage system mass, Eq. (5)
$\gamma$ = specific area of power supply, ft$^2$/kw
$\eta$ = efficiency
$\theta$ = angle between the normal to the solar paddle and the free stream velocity vector, rad
$\rho$ = density, $lb_m$/ft$^3$
$\sigma$ = Stefan-Boltzmann constant = $0.173 \times 10^{-8}$ Btu/ft$^2$-hr-°R$^4$
$\varphi$ = angle between the vertical and the support rod, rad
$\tau$ = thickness, in.

### *Subscripts*

$a$ = array
$b$ = battery
$c$ = gravitational constant, cold side of storage system
$D$ = drag
$cs$ = cross sectional
$e$ = exit
$f$ = frozen
$fcs$ = filtered cell in space
fo = foam
$g$ = acceleration forces
$h$ = hot side of storage system, horizontal direction
$l$ = liquid
$m$ = maximum
$ne$ = not evacuated
$o$ = stagnation
$O_2$ = oxygen
$p$ = power, propulsion
$pp$ = powerplant
$pp_i$ = incremental powerplant
pr = propellant
$pr_i$ = incremental propellant
$r$ = power regulation
ro = rod
$s$ = sunlit
$ss$ = storage system
$T$ = total
$t$ = time
ta = tank
$tu$ = ultimate
$ty$ = yield
$v$ = vertical
va = vapor
$w$ = weld
$\infty$ = freestream

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## Introduction

PROPONENTS of electric propulsion systems see considerable advantage in their use, assuming the availability of light weight power supplies. Opponents point out that such power supplies will not be available until the distant future. The present study considers the feasibility of an electric propulsion system using a photovoltaic power supply (making some allowance for improvement in the next 2 years) for attitude control and orbit maintenance for an MORL with an operating lifetime of 1 year. Minimum system total mass (power supply, propellant, and propellant storage system) is obtained using specific impulse values that fall in the range most efficiently attained with arc-jet engines. It is assumed that the MORL obtains its electric power for general use from a solar cell array, and the primary purpose of the attitude control system (consisting of momentum flywheels in addition to reaction control jets) is to provide solar orientation for this array. The vehicle is assumed to be nonrotating.

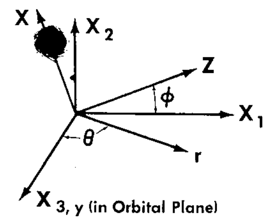
The total thrust requirement for the arc-jet propulsion system has been taken to be 0.10 $lb_f$, supplied continuously for one year (total impulse equal to $3.15 \times 10^6$ $lb_f$-sec). This is a nominal figure not applicable to any particular vehicle configuration, but it is a reasonable figure based on studies of a number of possible configurations, moment arms, and vehicle orientation schemes for vehicles whose mass in orbit is approximately 75,000 $lb_m$. This continuous thrust requirement implies a relatively large energy or angular momentum storage capability, since the reaction control system must be capable of operating while the vehicle is in the earth's shadow. For continuous thrust, the total mass of the reaction control system is nearly linear with total impulse; hence, the mass estimates given can be scaled for moderately larger or smaller requirements.

## General Considerations

The propulsive corrections for attitude control and orbit maintenance will depend upon the configuration selected for the space station and the orbital altitude. The MORL is assumed to be in a circular orbit at an altitude of 200 naut miles, with an orbital inclination of 28.5° to the earth's equatorial plane. The orbital period is 91.3 min with dark time ranging from 29.5 to 35.9 min depending on the time of year. The vehicle is oriented so that the $X$ axis is always pointed toward the sun and the $Y$ axis lies in the orbital plane (Fig. 1). The principal moments about the $X$ and $Z$ axes are approximately equal.

The equations that describe the gravity gradient torques about the three principal axes are shown in Fig. 2. The $X_1$, $X_2$, and $X_3$ axes are an orthogonal set with $X_3$ coincident with $Y$ and with $X_1$ and $X_3$ in the orbital plane. With this notation, $\varphi$ is a constant and $\theta$ undergoes a variation of $2\pi$ for each orbit.

The disturbance torques, which are assumed to be due primarily to the gravity gradient and only secondarily to aerodynamic forces, vary nearly sinusoidally with time, with a period equal to half the orbital period. The component of the disturbance torque along each of the principal axes may be divided into two time-dependent parts: 1) a part whose time-averaged value is zero; and 2) a part with constant algebraic sign or with a period much longer than the orbital period. The first part can be handled with momentum flywheels alternately accelerating and decelerating as the torque reverses direction. The second part, along with the thrust needed for orbit maintenance, establishes the requirements for the arc-jet propulsion system. If the reaction control system uses arc-jets, it is advantageous to use flywheels to store the disturbance momentum in the dark portion of the orbit and to unload these wheels in the sunlight. For the purposes of this study, the flywheels used to store the disturbance momentum in the dark portion are considered to be separate elements, and their associated mass is charged to the arc-jet system. The disturbance momentum would be handled, in an actual spacecraft, by properly sizing the momentum wheels that are included to handle part 1 of the disturbance torques. If the arc-jet system is handled in this manner, it will be charged only with the incremental mass of the flywheels.




$$T_x = \frac{\lambda}{2} \cos \phi \; (I_z - I_y) \sin 2\theta$$

$$T_y = -\frac{\lambda}{2} \sin 2\phi \; (I_x - I_z) \left( \frac{1}{2} + \frac{\cos 2\theta}{2} \right)$$

$$T_z = -\frac{\lambda}{2} \sin \phi \; (I_y - I_x) \sin 2\theta$$

**Fig. 2 Disturbance torque equations.**

In selecting a propellant for the arc-jet engine, the necessary stagnation temperature is a prime consideration. Stagnation temperature vs $I_{sp}$ is shown for various propellants[2] (Fig. 3). For the specific impulse of interest here, hydrogen is the only fuel that can operate with a reasonable stagnation temperature. The propellant storage properties for liquid hydrogen used in the calculation of the storage system weight are as follows: storage pressure, 5 atm; boiling point, 49.0°R; densities of saturated liquid and vapor, 3.77 and 0.40 $lb_m/ft^3$, respectively; and latent heat of vaporization, 158 Btu/$lb_m$.

The number of engines will be determined in part by the final MORL configuration. Two pairs of fixed engines on each principal axis will provide angular corrective torques without introducing translational propulsive thrust. For purposes of comparison with the chemical system it is assumed that the reaction control arc-jets will have the same moment arm as those suggested for chemical thrustors. Two additional gimbaled engines will be required to overcome the drag forces. For efficiency, the thrust axes of these engines must be aligned with the vehicle velocity vector. A mass of 5 $lb_m$/engine is assumed, along with a total of 100 $lb_m$ for the gimbaling hardware. It may be possible to place part of the solar cells on the outer skin of the vehicle rather than on paddles; this would eliminate some of the extra drag created by the solar paddle structure.

The question of the feasibility of the operation of an arc-jet engine for long periods of time is of concern for this mission. Since the satellite will be manned, occasional replacement of electrodes and nozzles is possible and probably

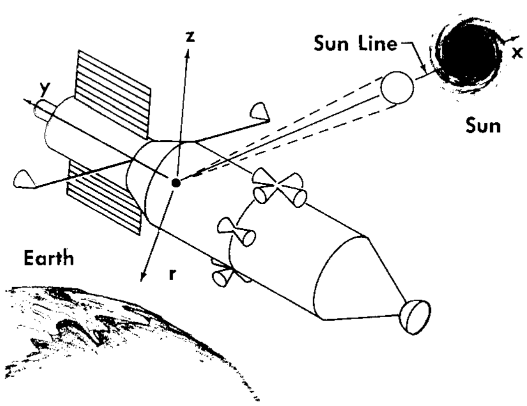



**Fig. 1 Orientation of MORL in orbit.**

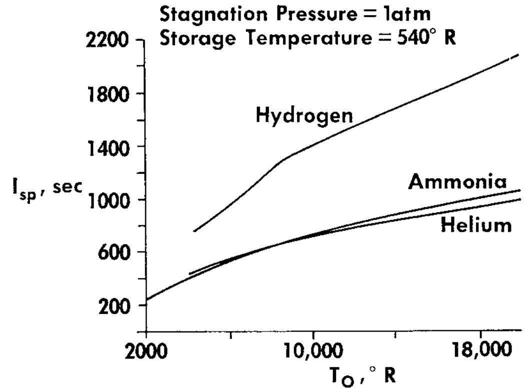



**Fig. 3 Specific impulse vs stagnation temperature for for $H_2$ $NH_3$, and $H_e$.**

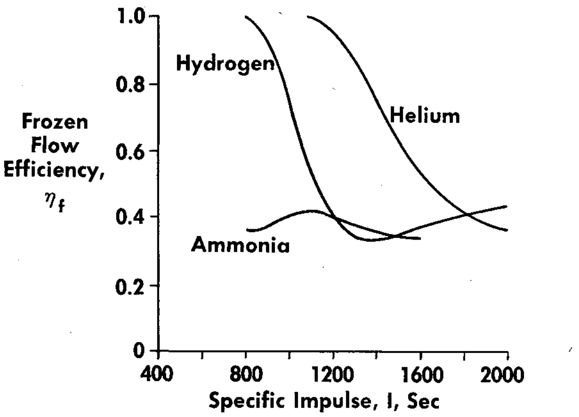



**Fig. 4 Frozen flow efficiency vs specific impulse for $H_2$, $NH_3$, and $H_e$.**

tolerable. Results recently published[3] concerning a development test of a 30-kw arc-jet engine indicate that such an engine was operated for 1 month without servicing or disassembling. The specific impulse during the test was 1000 sec. The mass losses for the anode and cathode were 0.01 and 0.7%, respectively, small enough to indicate that engine operation could have continued for a much longer time.

## Basic Performance Relations

If an expansion to zero pressure is assumed, all of the thermal energy of the propellant in the arc-jet engine is converted to directed kinetic energy except for the heat of formation of dissociated species present at the nozzle exit. It is then possible to write

$$H_o = V_e^2/2g_c\,\eta_f J = g_c\,I_{sp}^2/2\eta_f\,J \tag{1}$$

The actual expansion will be to some finite pressure at the nozzle exit, since finite area ratio nozzles will be used. For the small mass flow rates of interest here, however, the area ratio can be of the order of 100 and the loss in directed kinetic energy less than 1%. The frozen flow efficiency is defined as the ratio of thermal and kinetic energy in the exhaust jet to total energy put into the propellant $\eta_f = 1 - (H_f/H_o)$ where $H_f$ is enthalpy frozen in dissociation or ionization. Power supplied to the engine is then

$$P = 1.055\,\dot{m}_{pr}\,H_o/\eta_p = 1.055\,F\,H_o/I_{sp}\eta_p \tag{2}$$

where $\eta_p$ is the fraction of input energy reaching the propellant, 1.055 converts Btu/sec to kw, and initial enthalpy of the propellant has been neglected. The mass flow rate for a given thrust and $I_{sp}$ is $\dot{m} = F/I_{sp}$.

The mass of the powerplant $M_{pp}$ is based upon choice of a solar cell panel to provide electrical energy to create and maintain the arc used to heat the propellant gas. Using Eqs. (1) and (2),

$$M_{pp} = \alpha P = 1.055\alpha\,Fg_c\,I_{sp}/2J\,\eta_f\eta_p \tag{3}$$

where $\alpha$ is the specific mass of the power supply (includes power conditioning and power conversion equipment). Since the arc-jet engines are to be operated in the sunlit portion of the orbit, $F = F_c\,t_o/t_s$, where $F_c$ is the continuous thrust required when no momentum storage capability is included.

From the foregoing relations it is clear that total mass can be optimized with regard to $I_{sp}$. There is, however, another requirement. The drag on the vehicle due to the additional solar paddle required would not occur if a chemical propulsion system were used. To compensate for this additional drag force, the arc-jets must provide greater thrust for orbit maintenance than that required of a chemical propulsion system. The value of this incremental thrust, denoted $F_i$ and based on operation in sunlight, can be used to compute the incremental power supply mass and the incremental propellant mass. Total mass of the arc-jet propulsion system is given by

$$M_T = M_{pp} + M_{ppi} + (M_{pr} + M_{pri})\beta + C \tag{4}$$

where $C$ represents the mass of engines, momentum flywheels, and associated hardware, including the batteries and portion of the solar array required to provide energy to operate the momentum wheels in the dark portion of the orbit, and $\beta$ is a factor that allows for the propellant storage system mass

$$\beta \equiv (M_{pr} + M_{pri} + M_{ss})/(M_{pr} + M_{pri}) \tag{5}$$

where $M_{ss}$ is the mass of the storage system. Substituting for $M_{pp}$, $M_{ppi}$, $M_{pr}$, and $M_{pri}$ gives

$$M_T = 1.055\,\alpha g_c\,I_{sp}(F + F_i)/2J\eta_f\eta_p + \beta t_p(F + F_i)/I_{sp} + C \tag{6}$$

where $F_i = F_i(I_{sp})$. The incremental drag force created by the additional solar paddle area at any point in orbit may be approximated by

$$D_i = \rho V_\infty^2 C_D A_T\,|\cos\theta|/2g_c \tag{7}$$

The paddle is assumed to be oriented so that it always faces the sun, even when in the earth's shadow. Integrating (7) for a complete orbit, the mean drag force is then given by

$$\bar{D}_i = \rho V_\infty^2 C_D A_T/\pi g_c \tag{8}$$

The incremental thrust is greater than $\bar{D}_i$, since the arc-jets operate only in sunlight:

$$F_i = \bar{D}_i\,t_o/t_s \tag{9}$$

The total paddle area can be expressed in terms of the power and specific paddle area $A_T = (P + P_i)\gamma$. Using this with Eqs. (1–3),

$$A_T = 1.055\,\gamma(F + F_i)g_c I_{sp}/2\eta_f\eta_p J \tag{10}$$

Combining Eqs. (8–10), and letting

$$a \equiv 1.055\,\gamma\rho V_\infty^2\,C_D t_o/2\pi J t_s \tag{11}$$

gives

$$F_i = aI_{sp}(F + F_i)/\eta_f\eta_p \tag{12}$$

The total mass can be expressed as a function of the specific impulse by eliminating $(F + F_i)$ between Eqs. (12) and (6):

$$M_T = (I_{sp} + \eta_f\eta_p e/I_{sp})bF/(\eta_f\eta_p - aI_{sp}) + C \tag{13}$$

where

$$b \equiv 1.055\alpha g_c/2J \qquad \text{and} \qquad e \equiv \beta t_p/b \tag{14}$$

To find the $I_{sp}$ for minimum total mass, Eq. (13) is differ-

**Table 1 Numerical values for the parameters appearing in the optimization equation**

| Parameter | Value |
|---|---|
| $a$ Eq. (11) | $37.9 \times 10^{-6}$ $lb_m/lb_f$-sec |
| $b$ Eq. (14) | 3.55 $lb_m^2/lb_f^2$-sec |
| $\beta$ Eq. (5) | 1.41 |
| $e$ Eq. (14) | $7.60 \times 10^6$ $lb_f$-sec/$lb_m^2$ |
| $C$ Fixed mass, as follows: | |
| 14 engines | = 70 $lb_m$ |
| Gimbaling mounts | = 100 $lb_m$ |
| Flywheels | = 750 $lb_m$ |
| Batteries | = 100 $lb_m$ |
| Solar array for batteries and associated power cond. equip. | = 67 $lb_m$ |
| Tot. fixed mass, $C$ | = 1087 $lb_m$ |
| $\eta_f$ Fig. 4 | 0.846 |
| $\eta_p$ assumed | 0.60 |
| $I_{sp(opt)}$ Eq. (15) | 963 $lb_f$-sec/$lb_m$ |

entiated. Note that $a$, $b$, $e$, $F$, $\eta_p$, and $C$ are constants. Setting $dM_T/dI_{sp} = 0$ and simplifying,

$$I_{sp}^2\{1 - (I_{sp} + ae)(d\eta_f/dI_{sp})/\eta_f\} + 2aeI_{sp} - \eta_f\eta_p e = 0 \quad (15)$$

This expression is readily solved for $I_{sp}$ using a curve of $\eta_f$ vs $I_{sp}$ given in Ref. 2 and reproduced here as Fig. 4. For any trial value of $I_{sp}$, values of $\eta_f$ and $d\eta_f/dI_{sp}$ are obtained from the curve, and the lefthand side of (15) evaluated. The value of $I_{sp}$ satisfying (15) gives the minimum system mass. [There will, in general, be more than one value of $I_{sp}$ which satisfies (15), given a variation of $\eta_f$ with $I_{sp}$ similar to that shown in Fig. 4, but the authors have found no difficulty in establishing the solution giving lowest system total mass.] The values of $\alpha$, $\beta$, $\gamma$, and $C$ (developed hereinafter) are given in Table 1. Referring to Eq. (11), the definition of $a$ involves knowledge of $V_\infty^2$. The space station is specified as being in a 200 naut miles ($1.216 \times 10^6$ ft) circular orbit, hence (with $R = 2.067 \times 10^7$ ft),

$$V_\infty^2 = g_o R/(1 + h/R) = 6.281 \times 10^8 \text{ ft}^2/\text{sec}^2 \quad (16)$$

Using Eq. (15) with the quantities shown in Table 1, the resulting optimum specific impulse is 963 $\text{lb}_f$-sec/$\text{lb}_m$. This specific impulse is clearly a function of the choices made for the constants $a$, $b$, $e$, $F$, $\eta_p$.

Using this value of $I_{sp}$ and going back to Eq. (13) gives a system total mass of 7258 $\text{lb}_m$ distributed as follows: power supply, 1196; propellant, 3528; storage system (at $\beta = 1.41$), 1447; and fixed hardware ($C$), 1087. Note (Fig. 5) that for the choices of $a$, $F$, $\eta_p$, and $C$ made here (see Table 1), a power supply with a specific mass of 4 lb/kw still does not give rise to a specific impulse that is out of the range of applicability for the arc-jet, because drag penalty (unchanged) acts to reduce optimum $I_{sp}$. Furthermore, for a power supply that can be considered drag free (i.e., placed entirely within the confines of the space station without having to increase the station's volume to accomplish this) a specific mass of less than 50 lb/kw would be required to raise the optimum specific impulse to greater than 1200 $\text{lb}_f$-sec/$\text{lb}_m$.

## Power Supply System

A solar photovoltaic power supply was selected for this mission with mass and performance estimates based on current state of the art with reasonable extrapolations to the 1965 time period. The system is composed of a solar cell array, power conditioning equipment, and storage batteries (Fig. 6). The array must be made larger than the size needed

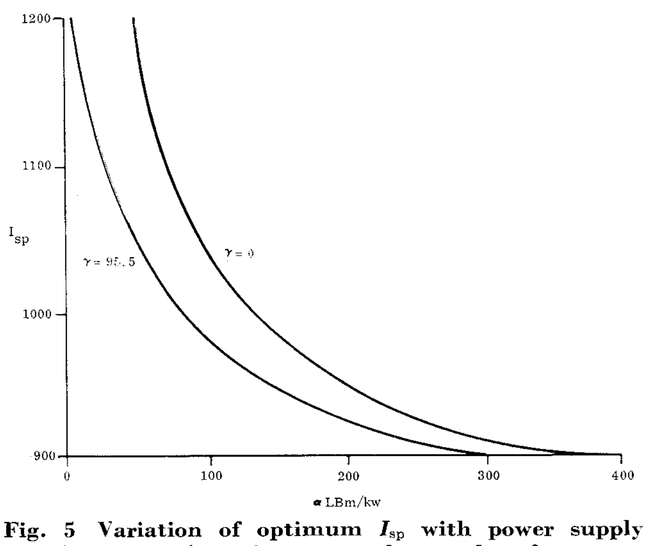



**Fig. 5 Variation of optimum $I_{sp}$ with power supply specific mass $\alpha$ ($\gamma = 0$ corresponds to a drag free power supply.)**

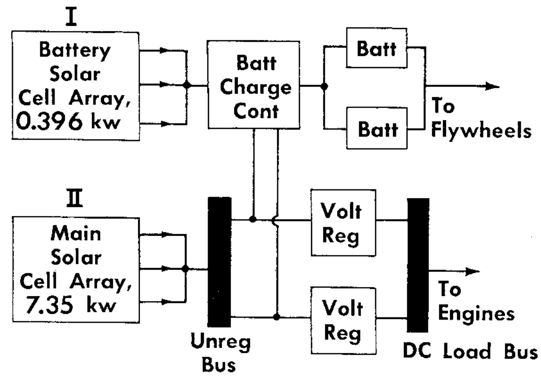



**Fig. 6 Schematic representation of power supply system.**

by the arc-jets to provide continuous thrust. Energy must also be supplied to the arc-jets for additional thrust to unload the flywheels in the sunlit portion of the orbit. A small amount of energy must also be supplied to charge batteries which in turn supply energy for operation of the flywheel motors in the dark portion of the orbit.

The solar cell array consists of oriented paddles, on one side of which are mounted $1 \times 2$ cm gridded $N/P$ solar cells with cover glass of 6 mil thickness. At present, the mass of solar paddle structure and hardware is estimated as 0.86 $\text{lb}_m$/ft$^2$. The mass of solar cells, cover glass, wire diodes, and adhesives is approximately 0.53 $\text{lb}_m$/ft$^2$. This yields an array mass of 1.39 $\text{lb}_m$/ft$^2$. The specific area of the solar cell array is given by

$$\gamma = (C_p C_t E \eta_{fcs})^{-1} = 95.5 \text{ ft}^2/\text{kw} \quad (17)$$

assuming values for packing factor $C_p = 0.87$, solar radiation intensity $E = 0.130$ kw/ft$^2$, efficiency for filtered cell in space $\eta_{fcs} = 0.11$, and solar cell temperature correction factor at 150°C, $C_t = 0.84$. The specific mass of the solar array is then 132.7 $\text{lb}_m$/kw.

Assuming that the power conditioning equipment has a specific mass $\alpha_r = 11$ $\text{lb}_m$/kw (which will be doubled for reliability) and an efficiency $\eta_r$ of 0.95, total specific mass of the power supply is

$$\alpha = (\alpha_a + \alpha_r)/\eta_r = 162.8 \text{ lb}_m/\text{kw} \quad (18)$$

Energy required from the batteries to operate the flywheels in the dark portion is only 161 w-hr/orbit. Nickel-cadmium batteries are chosen for this study. Their specific energy varies from 8–14 w-hr/$\text{lb}_m$ depending on discharge rate. This figure includes the case weight and related hardware; the lower figure is used to obtain a conservative estimate of the battery specific mass. For the lifetime and orbital period characteristics of this mission, 5750 cycles of operation are required. From an experimental curve of depth of discharge vs cycle life, a 40% depth of discharge appears attainable. Two batteries are included to achieve a high reliability. The battery mass is then

$$M_b = 2 \times 161/0.40 \times 8 = 100 \text{ lb}_m$$

To recharge the batteries in sunlight, making conservative allowance for losses in the circuitry required, the array must be increased to provide an additional 0.396 kw. The associated mass is 64 $\text{lb}_m$. Allowing 3 $\text{lb}_m$ for a battery charge controller, the total power supply mass for operation of the momentum flywheels is 167 $\text{lb}_m$.

## Storage System

The suggested storage system utilizes a spherical tank within an imaginary cubic lattice so that the tank is supported by titanium alloy connecting rods from points on the sphere in contact with four of the faces of the cube (Fig.

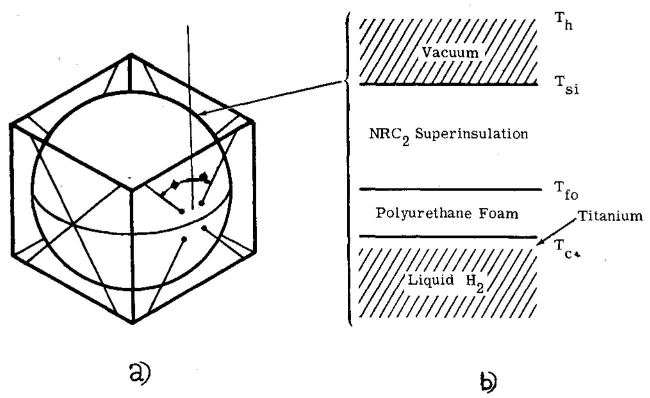



**Fig. 7 Cryogenic storage system: a) storage tank and support rod configuration and b) cross-sectional view of storage tank and insulation layers.**

7a). The tank and support rods are enclosed in NRC-2 superinsulation. The storage system will contain the storage tank, support rods, and insulation, a phase separating venting system, baffles that might also serve to position the liquid vapor interface,[4] an ohmic heating coil, a delivery line from the inner shell through the insulation, a separate fill line, and a quantity gage. It is assumed that the storage system will be placed within the confines of the space station so that the temperature external to the insulation is that of the space station environment (520°R). This portion of the space station is permitted to outgas to space in order that the insulation can be effective. Figure 7b is a schematic cross-sectional view of the storage tank. The thickness of the inner shell is calculated as the sum of the thickness required from burst pressure considerations and the thickness required to withstand the inertia load of the fluid in the tank during launch. The volume of the storage tank is

$$V = 4\pi r_{ta}{}^3/3 = (M_{pr} + M_{pri})/\rho_1 \qquad (19)$$

where

$$M_{pr} + M_{pri} = Ft_p/[I_{sp}(1 - aI_{sp}/\eta_f\eta_p)] = (0.165)(19.1 \times 10^6)/\{963[1 - (37.8 \times 10^6) \times (963)/(0.846)(0.60)]\} = 3528 \text{ lb}_m \qquad (20)$$

Solving (19) for the radius of the storage tank yields

$$r_{ta} = [(3)(3528)/(4\pi)(3.77)]^{1/3} = 6.068 \text{ ft} \qquad (21)$$

Following the procedure of Ref. 4, the thickness based on burst pressure is calculated using the thin spherical shell formula of Svenson[5]:

$$\tau_p = P_m S(1.5)^n r_{ta}/2\eta_w F_{tu} \qquad (22)$$

The spherical shell is assumed to consist of two hemispheres joined in a plane perpendicular to the vertical axis of the launch vehicle. There may be a flange or some overlap where the hemispheres meet, forming a fairly stiff ring. The launch acceleration forces are transmitted to the shell at four equidistant attachment points on this ring. Assuming that the ring transmits this load uniformly to the shell, the incremental shell thickness required for the acceleration loading, considering only the mass of the propellant, is given by

$$\tau_g = 2a_m\rho_1 S r_{ta}{}^2/3g_c F_{ty} \qquad (23)$$

The sum of (22) and (23) gives the shell thickness

$$\tau_p + \tau_g = \{(73.5)(2.25)(6.068)(1.5)^{0.113}/(2)(1.0) \times (1.40 \times 10^5)\} + (2)\ (31.623)(32.2 \text{ ft/sec}^2) \times (3.77)(2.25)(6.068)^2/(3)(32.2)(1.30 \times 10^5 \text{ lb}_f/\text{in.}^2) \times (144 \text{ in.}^2/\text{ft}^2) = 0.0041 \text{ ft}$$

The value of 2.25 for $S$ is thought to be reasonable for titanium alloys, the properties of which are given in Table 2. A foam jacket is placed on the inner shell to prevent condensation of atmospheric constituents during the prelaunch and launch ascent time. It is assumed that the vessel can be topped off one minute before launch. The foam will also reduce the boiloff during the minute of boost through the atmosphere and during the outgassing phase until the inner layers of superinsulation become effective.

The support rods are effective in tension only. There are, at any instant of time, only eight rods supporting the vertical load and four rods supporting each of the transverse loads. Referring to Fig. 7a we have

$$8f_v \cos\varphi = a_v\beta(M_{pr} + M_{pri})/g_c \qquad (24)$$

$$4f_h \sin\varphi = a_h\beta(M_{pr} + M_{pri})/g_c \qquad (25)$$

Some of the rods may be supporting a combined vertical and horizontal load given by

$$f_T = (\beta)(M_{pr} + M_{pri})(a_v/\cos\varphi + 2a_h/\sin\varphi)/8g_c \qquad (26)$$

Using this expression, the minimum value of $f_T$ occurs when $\tan\varphi = (2a_h/a_v)^{1/3}$. If the acceleration in the vertical direction is taken to be 30 $g$'s and that in the horizontal direction 10 $g$'s, $\varphi$ is found to be 41.14°, and $f_T = 43{,}676$ lb$_f$. The total load results from inertia of the propellant and the storage tank. The cross-sectional area of a support rod $A_{cs}$ is $f_T S/F_{ty} = 0.678$ in.$^2$ (see Table 2), hence $r_{ro} = 0.0387$ ft. The length of the conduction path for each rod is $r_{ta}/\cos\varphi = 8.06$ ft.

An approach to insure delivery of vapor only to the engines and to provide for maximum thermal energy input to the vented fluid has been suggested.[6] Fluid for the engines is passed through a throttling valve to a pressure lower than tank pressure. If any liquid is present, some of it will flash to vapor and the temperature of the fluid after throttling is lower than the temperature of the stored fluid. This fluid that has been throttled is then passed through a heat exchanger within the storage space permitting a transfer of energy from the stored fluid to the fluid that was originally vented from it. This transfer of energy assures that heat leakage into the tank is removed with the propellant being fed to the engines. The heat exchanger is supplemented with an electrical resistance heater to assure that the propellant is supplied to the engines in the vapor phase.

An amount of insulation is provided so that the total heat leakage to the stored propellant is one-half of that required to boil off the propellant at the use rate for the mission. The additional thermal energy will be provided by the electrical resistance heater. It is not necessary to maintain a constant storage pressure; it may be desirable to allow the pressure to drop as the propellant is being used in the sunlit portion of the orbit. During the dark time, when there is no demand for propellant, the structure is designed to permit the insulation and supports to increase the pressure to a maximum of 5 atm. The amount of heat required per unit

**Table 2 Physical properties of the titanium alloys chosen for the storage tank and support rods**

| Titanium 6 AL–4V(shell) | Titanium 4 AL–4 *Mn* (support rods) |
|---|---|
| $F_{tu}$ = 1.40 × 10$^5$ psi at 520°R; 2.67 × 10$^5$ psi at 37°R | $F_{ty}$ = 1.45 × 10$^5$ psi |
| | $k$ = 3.5 Btu/hr-ft-°R |
| $F_{ty}$ = 1.30 × 10$^5$ psi at 520°R; 2.60 × 10$^5$ psi at 37°R | $\rho_{ro}$ = 281 lb/ft$^3$ |
| $\eta_w$ = 0.90 | |
| $n$ = 0.113 | |
| $\rho_{ta}$ = 276 lb/ft$^3$ | |

time to boil off propellant is given by

$$\left.\begin{aligned}\dot{Q} &= h_{fg}\dot{m}\rho_1/(\rho_1 - \rho_{va}) = 158\ (1.04 \times 10^{-4})(3.77)/3.37 \\ &= 0.0184\ \text{Btu/sec} \\ &= 66.1\ \text{Btu/hr}\end{aligned}\right\} \quad (27)$$

The factor $\rho_1/(\rho_1 - \rho_{va})$ is included to allow for increase in the volume of vapor in the tank with time. Heat leak through each support rod due to conduction and radiation may be approximated by the relation

$$\dot{q}/\text{rod} = k_{ro}\pi r_{ro}^2(T_h - T_c)\cos\varphi/r_{ta} + 0.005(2\pi r_{ro})\alpha(T_h^4 - T_c^4) = 3.5\pi(0.0387)^2 \times (520 - 49)(0.7531)/6.068 + 0.010\pi(0.0387) \times (0.173 \times 10^{-8})[(520)^4 - (49)^4] = 1.12\ \text{Btu/hr} \quad (28)$$

where 0.005 is an empirical constant in feet. The second term accounts for radiation via multiple reflections within the inner layers of insulation from the hot end to the cold end. This is highly dependent on wrapping technique and is not wholly understood at this time. The heat leak allowed is

$$\dot{q}_{si} = \dot{q}_{fo} = \dot{Q}/2 - 16\dot{q}/\text{rod} = 15.2\ \text{Btu/hr} \quad (29)$$

The thickness of the foam layer and superinsulation provided can be calculated assuming steady state conductive heat transfer. For this we require the cryogenic vessel to be filled and continuously topped before launch. Assuming the foam layer and superinsulation thickness to be much smaller than the radius of the inner vessel,

$$\begin{aligned}\tau_{si} &= -4\pi(k_{si})r_{ta}^2(T_c - T_h)/\dot{q}_{si} \\ &= -4\pi(2.5 \times 10^{-5})(6.068)^2(49 - 520)/15.2 = 0.359\ \text{ft}\end{aligned} \quad (30)$$

$$\begin{aligned}\tau_{fo} &= \tau_{si}k_{fo}(T_c - T_{o_2})/(k_{si})_{ne}(T_c - T_h) \\ &= (0.359)(0.0217)(49.0 - 162.3)/(0.0148)(49 - 520) \\ &= 0.127\ \text{ft}\end{aligned} \quad (31)$$

The masses of the titanium tank, support rods, superinsulation, and foam can be calculated by computing the total volume of material and multiplying by its density. Density of the foam is 2.0 lb/ft$^3$ and density of the superinsulation is 2.4 lb/ft$^3$.

$$\begin{aligned}M_{\text{Ti(tank)}} &= 1.3\ (4\pi r_{ta}^2)\tau_t\rho_{ta} \\ &= 5.2\pi(6.068)^2(0.0041)(276) = 681\ \text{lb}_m\end{aligned} \quad (32)$$

The factor 1.3 is included to account for nonuniformities in thickness of the spherical shell and extra thickness required for increased stresses at attachment points and weld joints. Similarly,

$$M_{Ti(\text{rods})} = N\pi r_{ro}^2\rho_{ro}r_{ta}/\cos\varphi = 170\ \text{lb}_m \quad (33)$$

where $N$ = 16 rods, and $\rho_{ro} = 281\ \text{lb}_m/\text{ft}^3$

$$M_{si} = 4\pi r_{ta}^2\tau_{si}\rho_{si} = 399\ \text{lb}_m \quad (34)$$

$$M_{fo} = 4\pi r_{ta}^2\tau_{fo}\rho_{fo} = 117\ \text{lb}_m \quad (35)$$

The total mass of the storage system is the sum of the foregoing four masses to which an additional 125 $\text{lb}_m$ is added to account for feed lines, valves, solenoids, and associated hardware.

$$M_{ss} = M_{Ti(\text{tank})} + M_{Ti(\text{rods})} + M_{si} + M_{fo} + 125 = 1492\ \text{lb}_m \quad (36)$$

Using Eq. (5) yields $\beta = 1.423$ which is close enough to the original value of 1.41 used in the system total mass calculation.

## Momentum Flywheels

The momentum flywheels are assumed to be at rest as the space station enters the dark portion of the orbit, and a constant torque (2 ft-$\text{lb}_f$) is then assumed to be applied to accelerate the wheels continuously to a maximum velocity of 280 rpm. This represents the saturation velocity for the wheels reached at the end of the dark portion. The calculations assume that the wheels are unloaded in the same amount of time it takes to load them and thus there will be a short time interval during which the wheels will not be rotating. Wheels, 11 ft in diameter, each having a rim mass of 156 lb could provide the required storage capability.[7] The mass of each wheel is calculated by equating the kinetic energy of each, when rotating at maximum speed, with the energy needed to saturate it. No attempt is made to calculate the fraction of the total energy provided to each; rather each is sized to handle the total.

The wheels can be placed either outside the craft or within an evacuated section. Brushless d.c. motors to torque the wheels can be placed in a chamber with labyrinth shields at the hub. The heat generated by the stator and rotor may be transferred via the hub directly to the space station where it is assumed to be handled by the space vehicle environmental control system. To account for the mass of motors, shields, spokes, lubrication materials, and related hardware, a mass of approximately 94 lb/wheel is added to the previous amount. The mass for each flywheel "system" then, is 250 $\text{lb}_m$, giving a 750 $\text{lb}_m$ total for all three. The breakdown of the total fixed mass $C$ is shown in Table 1.

## Chemical Propulsion System

The chemical bipropellant system used as a basis for comparison with the arc-jet system is outlined in Table 3. The thrust of the chemical system can be made large enough (with negligible fixed mass penalty) to allow for orbit maintenance by firing the proper attitude control jets at points in orbit for which the thrust axis of the jets is aligned with the velocity vector of the vehicle. This enables the chemical system to provide attitude control and orbit maintenance with just 12 engines and eliminates the need for two gimbaled engines.

With intermittent operation, there will be poor performance at the beginning and end of each pulse. No allowance has been made for this in the system mass calculation.

**Table 3 Chemical bipropellant system assumed for comparison with arc-jet system**

| Item | Value |
|---|---|
| Delivered $I_{sp}$ (continuous operation) | 300 sec |
| Mixture ratio, $0/F$ | 2:1 |
| Nozzle area ratio, $A_e/A_t$ | 40:1 |
| Characteristic chamber length, $L^*$ | 10 in. |
| Chamber pressure | 100 psia |
| Propellant tank pressure | 200 psia |
| Gas tank pressure | 3000 psia |
| Pressurizing gas | Helium |
| Tankage material | (6 AL–4 V) |
| $F_{ty}$ | 16,000 psi UTS |
| Weight breakdown ($\text{lb}_m$) | |
| Oxidant, $N_2O_4$ | 7370 |
| Fuel 50% UDMH/50% $N_2H_4$ | 3690 |
| Gas pressurant, helium | 40 |
| Tanks | |
| Oxidant | 170 |
| Fuel | 130 |
| Pressurant | 270 |
| 12 thrustors | 40 |
| Valves and lines | 80 |
| Total system mass (12 nozzles) | 11,790 $\text{lb}_m$ |

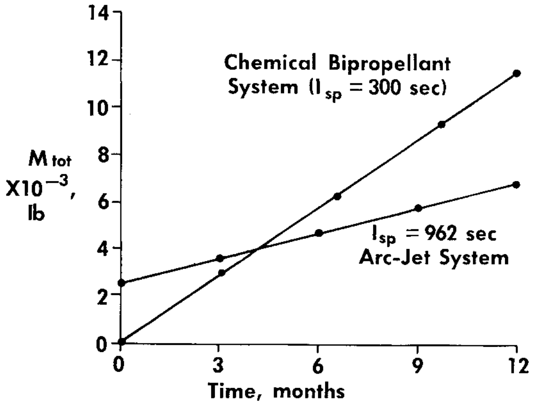



**Fig. 8 System total mass vs lifetime.**

## Conclusions

This study indicates that a solar-cell powered arc-jet reaction control system utilizing current state-of-the-art technology can provide attitude control and orbit maintenance for an MORL with a significant launch mass saving as compared to a chemical propulsion system. For the 1-yr mission, an arc-jet system having a specific impulse of 963 $lb_f/lb_m$-sec can provide a launch mass saving of 4532 $lb_m$ as compared to a chemical system with a specific impulse of 300 $lb_f/lb_m$-sec. This represents a 38% reduction indicating that consideration of the arc-jet system for this mission is warranted. For shorter missions, the crossover point at which the arc-jet system becomes lighter than the chemical system occurs between 3 and 4 months, assuming constant $I_{sp}$ (rather than optimum). System total mass vs lifetime is compared in Fig. 8.

The application selected appears to be particularly favorable for this type of electric propulsion system. The size of the propulsion system is large enough to make the launch mass saving attractive. The disturbance torques are of a low level continuous nature, arising primarily from gravity gradient and aerodynamic forces. These requirements as well as those for orbit maintenance can be satisfied with low thrust arc-jet engines. The additional solar-cell array required to power the arc-jets will be properly oriented along with the array used to provide power for general purposes. Thus, the additional array will present no additional constraint on vehicle orientation. Continual use of propellant during each orbit simplifies the cryogenic storage problem incurred in using hydrogen as propellant. Also, the low chamber pressures for the arc-jet as compared with the chemical rocket combustion chambers permits lower propellant storage pressures with concomitant savings in the thickness and mass of the storage tank shell. Presence of personnel in the spacecraft mitigates the reliability and maintainability problems.

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